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Granular Phononic Crystals as Tunable Functional Switches

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Abstract

In this work, we discuss a strategy for spatial and modal wave manipulation in granular phononic crystals based on the use of nonlinearity as a trigger to reversibly activate different combinations of wave modes in the crystals response. Our approach revolves around the concept of modal mixing, whereby, through the generation of higher harmonics in crystals with complex modal structures, we can induce jumps in the response across the available propagation modes; as a result, the system experiences a blend of modes and the simultaneous activation of complementary functionalities. To demonstrate the versatility of this approach, we numerically study a family of dimer granular crystal configurations featuring a variety of wave control functionalities. This approach based on modal mixing features the ability to yield a wide variety of functional configurations without changes in the shape, size or topology of the nonlinear phononic crystal.

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1. Introduction

Granular Phononic Crystals (GPCs) are periodic repetitions of unit cells where the inter- and intra-cell interactions are characterized by contact mechanisms. Since the contact interactions are intrinsically nonlinear, GPCs can be tuned to exhibit a wide range of system responses ranging from linear to strongly nonlinear. Among the peculiar phenomena observed in GPCs are solitary waves [1, 2], discrete breathers [3] and modulational instability; these properties can be exploited for a variety of applications such as shock protectors [4, 5], acoustic lenses [6, 7], rectifiers [8], etc.
In the linear regime, GPCs exhibit frequency-dependent phononic characteristics, i.e., their ability to propagate or attenuate waves and their spatial wavefield characteristics are dependent on the frequency of the excitation [9, 10]. As we transition into the nonlinear regime, the spectral and spatial phononic characteristics of propagating waves are also dependent upon the amplitude of the excitation and, consequently, upon the external forces acting on the system. This additional tunability of the *spectral* characteristics has been utilized to design tunable vibration filters [11, 12].
In contrast, the effect of nonlinearity on the *spatial* characteristics of propagating excitations has been only marginally explored. In the weakly nonlinear regime, the effect of nonlinearity manifests as a modulation of the envelope of the propagating excitation [13]. For spring-mass chains with quadratic nonlinearity, this modulation is described by the Nonlinear Schrödinger equation [14, 15]. This long-wavelength modulation is also observed for more complex

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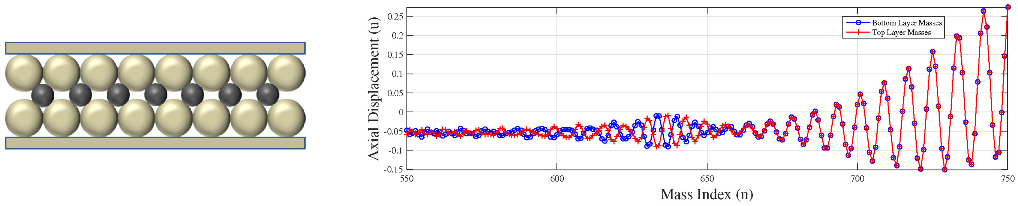

Fig. 1: (a) Schematic of granular waveguide; (b) Spatial wave profile in the top and bottom layers of the waveguide corresponding to the region of coexistence of the acoustic and optical modes.

granular chains and can be used to inversely characterize their nonlinear mechanisms [16].
Another effect of nonlinearity is the generation of super- and sub-harmonics in the response due to the self-interaction of the applied excitation. In dispersive systems (characterized by multiple folding branches in the dispersion relation), the ability to propagate higher harmonics is governed by the availability of dispersion branches in the frequency range of the activated harmonics [17, 18]. Moreover, the spatial characteristics of the activated harmonics also conform to the modal characteristics of the inherent linear system. This concept was recently studied from the perspective of tunable nonlinear phononic crystals in [19]. For nonlinear spring-mass chains, a multiple-scale expansion of the equations of motion reveals that the homogeneous equation at every order of expansion remains the same and that the nonlinear terms can be treated as forcing functions [20]. Therefore, the higher order solutions are indeed a linear combination of the mode shapes of the homogeneous linear eigenvalue problem. In this work, we will specialize and extend the paradigm proposed in [19] to GPCs with rectangular lattice structures. We will first briefly revisit the case of a 1D waveguide in a granular context and we will use this result as a stepping stone into the investigation of 2D crystals with tunable directivity.

2. Tunable Granular Waveguide

Let us consider a 1-D granular truss consisting of two monoatomic chains made of PTFE beads interconnected by a heavier set of Aluminum beads (shown in fig. 1(a)). The diameter of the heavier beads is chosen such that they act as internal resonators. Each bead is assumed to have one (horizontal) degree-of-freedom and a precompression force is applied in the horizontal direction to ensure that the beads remain in contact. The equations of motion are numerically integrated to determine the spatiotemporal response to a 7-cycle sine-burst modulated by a slowly varying Hann-window envelope, whose carrier frequency is determined from the linearized dispersion relation.
When the amplitude of the excitation is comparable to the initial precompression, we observe propagation of second harmonics in the response. For an excitation frequency belonging to the acoustic branch, the second harmonic *hops* from the acoustic to the optical mode. In the wave profile plotted in fig. 1(b) (bottom and top layers), we observe the coexistence of oscillatory and non-oscillatory features. The non-oscillatory feature is attributed to the modulation of the envelope due to nonlinearity. In addition, the packet notably features two distinct oscillatory components with contrasting inter-layer dynamics: the top and bottom layers exhibit in-phase dynamics in the dominant feature and out-of-phase dynamics in the secondary oscillatory component. These contrasting features are indeed consistent with the eigen mode shapes corresponding to the acoustic and optical branches (approximately determined from the eigen-vectors of the linear model). Therefore, we conclude that mode hopping due to harmonic generation yields an output signal with a broader frequency spectrum, which blends the deformation characteristics of multiple modes (*modal mixing*) and provides access to deformation patterns and modal attributes that are typical of the optical range, even while functioning in the low-frequency regime. This nonlinear waveguide can be envisaged as an amplitude-controlled switch between two operating modalities: one predominantly characterized by axial longitudinal deformation, the other triggering a partial energy migration to a slower mode that shears the structure horizontally.

3. 2D Granular Phononic Crystal with Tunable Directional Characteristics

We consider the classical rectangular close-packed 2D dimer GPC depicted in fig. 2(a), in which the bigger beads are made of Aluminum while the interstitial beads are made of Steel, and the system is precompressed equally in the horizontal and vertical directions. This configuration can also be seen as a 2D extension of the truss structure

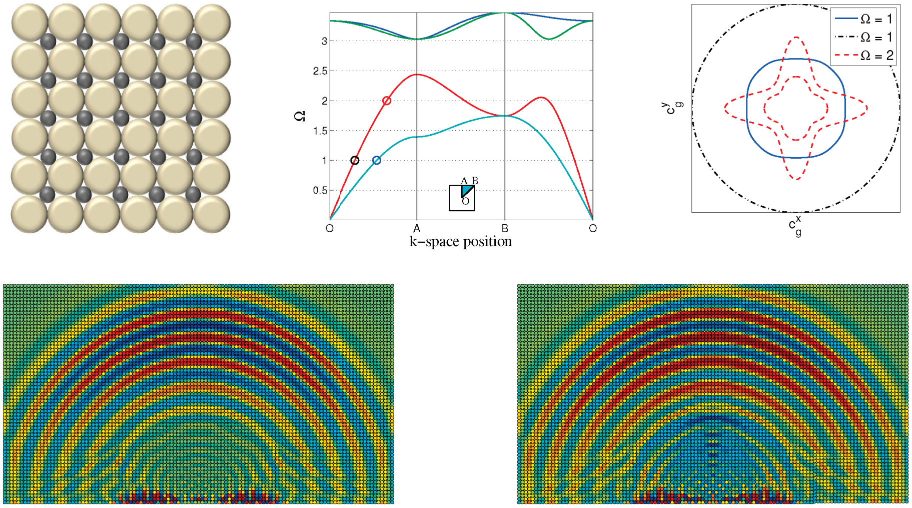

Fig. 2: (a) Schematic of 2D Rectangular Close-Packed Granular Lattice; (b) Linearized dispersion relation along the irreducible Brillouin zone *OABO*; (c) Group Velocity contours for two different frequencies (d-e) Wavefield established in the lattice (radial component is plotted) for two different amplitudes of excitation.

in fig. 1(a) upon selection of the appropriate dimension of the interstitial bead. The modal structure of the crystal can be fully captured by sampling the phase constant surfaces along the contour of the irreducible Brillouin zone (*OABO*), as depicted in the band diagram of fig. 2(b), where we observe the presence of two acoustic modes for low-frequency excitations. In order to determine the directional behavior of these modes, we plot the group velocity contours corresponding to two different frequencies ($\Omega = 1, 2$) in fig. 2(c). These frequencies have been chosen such that the higher excitation activates only a single mode. For the lower frequency, the group velocity contours suggest predominantly isotropic motion. In order to understand the spatial manifestation of these directionality contours, we excite the full-scale lattice structure vertically at the bottom and monitor the radial and tangential components of the displacement for every bead. Fig. 2(d) depicts the radial component of the spatial wavefield for an amplitude much smaller than the precompression and displays a circular wave profile.

As the amplitude is increased, we excite a second harmonic that lies on the longitudinal branch in a frequency range that is above the cutoff of the shear branch. The group velocity contours for the longitudinal mode at this frequency show orthotropic features. This is seen in fig. 2(e), which indeed reveals an additional feature that is oriented predominantly along the y direction. Therefore, by triggering the second harmonic, we change the modal mixture of the response in a way that globally favors longitudinal mechanisms and also modify the directivity of the crystal by activating new paths that are normally experienced by the crystal only at higher frequencies.

Another interesting feature is the possibility to tune by precompression, the characteristics of the nonlinearly-generated higher harmonics. The linearized dispersion relations for two different conditions of initial precompression ($Fx < Fy$ and $Fx > Fy$ keeping Fx fixed) is shown in figs. 3(a) and (b). It can be noticed that changing precompression results in profound modifications of the band diagram, namely a shift of the center and width of the bandgap. Let us assume to excite a wave of frequency ω in both cases: we can see that the harmonic (2ω) will either have propagating or attenuating characteristics depending upon the ratio of the precompression forces. Therefore, by merely changing the external forces, we can choose to divert energy into a non-propagating mode or impart negative-group velocity characteristics to the propagating wavefields.

4. Conclusion

We have proposed a strategy to manipulate spatial and modal characteristics of wave propagation in granular phononic crystals using the concepts of mode hopping and modal mixing. The reversible activation of functionality switching can be obtained as a spontaneous response to changes in the amplitude of excitation or by controlling external load parameters of the GPC. The topological complexity of the crystal, in conjunction with the material

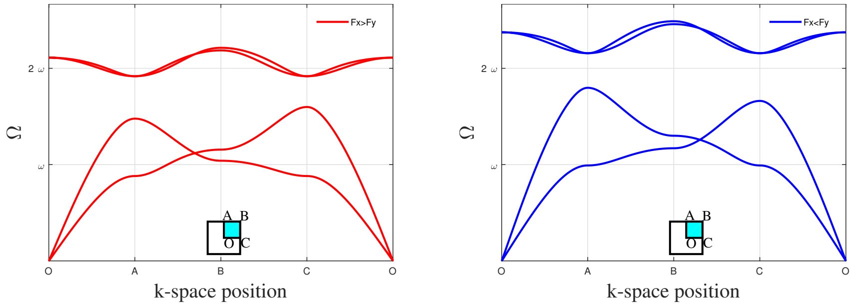

Fig. 3: (a-b) Linearized dispersion relation for two different values of the precompression in the horizontal and vertical direction along the irreducible Brillouin zone *OABCO*. The force in the horizontal direction is kept constant while the force in the vertical direction is varied.

properties, ultimately determines the landscape of functionalities that can be activated, thereby enabling access to virtually endless opportunities to engineer adaptive and tunable functional switches.

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